

AERODYNAMIC ANALYSIS OF GROOVE BASED VORTEX TRAP  
CONFIGURATION ON THE RETREATING HELICOPTER BLADE

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## ABSTRACT

The issue of high angle of attacks at the retreating blade receives attention from countless researchers because it is an important factor in the stability of the helicopter. The retreating blade stall will effect the stability of the helicopter due to the high angle of attacks. Prouty's helicopter data are used in this research to study the influence of the groove in the vortex trapped method on the helicopter stability. The method is implemented to control the flow of separation at the rotor blade on the retreating side of the rotor disc in forward flight conditions. One of the codes used to implement the method is the Blade Element Theory Code using Prouty's dataset. The code aids in the analysis of the aerodynamic characteristics of the helicopter main rotor blade. To validate the code, the angle of attack distribution along the main rotor blade in forward flight condition is compared to published data of Prouty's. Another approach is the use of Computational Fluid Dynamics to investigate the flow characteristics and aerodynamic parameters of the baseline helicopter aerofoil and modified helicopter aerofoil with the groove. The data obtained from both approaches are then combined in the analysis of the aerodynamic performance of both main rotor blades with or without the groove in forward flight condition. The influence of groove as a vortex trap is studied by means of the change in lift produced by the blade. The groove configuration is varied to determine its most suitable location, which is the critical sub-area at the retreating blade side, between  $0.6R$  to  $1.0R$  from the rotor hub, where  $R$  is the rotor disc radius. It is also found that the groove's location on the blade influences the choice of size. The best groove configuration is at  $0.5c$  from the leading edge, with the size of  $0.05c$ . The presence of groove affects the lift coefficient as simulated at  $0^\circ$  to  $18^\circ$  blade angle of attack. It is proven that the groove contributes significant aerodynamic capability in controlling the flow separation at high angle of attack. The important implication of this research is that the groove equipped helicopter blade offers an improvement in aerodynamic stability in which the load difference is 2.2% lower than that in the case of baseline blade. The blade also provides extra safety margin where the maximum angle of attack allowed is extra 6 degrees higher.

## ABSTRAK

Sudut serang yang tinggi pada bilah undur mendapat perhatian kebanyakan penyelidik, sebagai faktor penting dalam kestabilan sesebuah helikopter. Fenomena tegun bilah undur akan memberi kesan kepada kestabilan helikopter akibat sudut serang yang tinggi. Dalam kajian ini, data helikopter Prouty digunakan bagi mengkaji kesan alur dalam kaedah perangkat vorteks terhadap kestabilan helikopter. Kaedah ini bertujuan mengawal aliran terpisah pada bilah undur helikopter dalam penerbangan ke depan. Satu pendekatan yang digunakan adalah Kod Teori Unsur Bilah yang menggunakan input daripada Prouty. Kod ini membantu dalam analisa ciri-ciri aerodinamik bilah utama helikopter. Bagi menentusahkan kod ini, keputusan taburan sudut serang bilah utama daripada kod ini dibandingkan dengan data output Prouty. Satu lagi pendekatan adalah penggunaan perkomputeran dinamik bendalir bagi mengkaji ciri-ciri aliran dan parameter-parameter aerodinamik bilah piawai, dan bilah yang dilengkapi alur perangkat vortex. Data yang diperolehi daripada kedua-dua kaedah tersebut digabungkan dalam kajian prestasi aerodinamik kedua-dua bilah utama helikopter, samada dilengkapi dengan alur perangkat vorteks atau sebaliknya, dalam penerbangan ke depan. Kesan alur perangkat vorteks dikaji dengan menentukan selisih daya angkat yang dihasilkan oleh kedua-dua bilah. Variasi konfigurasi perangkat vorteks dikaji untuk menentukan lokasi alur yang sesuai; didapati lokasi kritikal di antara  $0.6R$  hingga  $1.0R$  dari hab rotor, di mana  $R$  adalah radius bilah. Turut didapati bahawa lokasi alur mempengaruhi saiznya. Konfigurasi alur yang terbaik adalah pada  $0.5c$  dari tepi depan airfoil dengan saiz  $0.05c$ . Didapati alur memberi kesan kepada pekali daya angkat pada sudut serang bilah antara  $0^\circ$  hingga  $18^\circ$ . Turut dibuktikan bahawa alur tersebut menyumbang kepada keupayaan mengawal aliran terpisah pada sudut serang yang tinggi. Implikasi penting kajian adalah peningkatan kestabilan aerodinamik bilah helikopter, di mana selisih beban pada kedua-dua bilah adalah 2.2% lebih rendah daripada selisih beban bilah helikopter piawai. Alur pada bilah juga meningkatkan margin keselamatan, di mana sudut serang maksimum yang dibenarkan tambahan 6 darjah lebih tinggi.



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## LIST OF SYMBOLS AND ABBREVIATIONS

SAR	Search and Rescue
GA	General Aviation
ASI	AOPA Air Safety Institute
IHST	International Helicopter Safety Team
FAA	Federal Aviation Administration
AAIB	Air accident investigation Branch
EHEST	European Helicopter Safety Team
BET	Blade Element Theory
BETC	Blade Element Theory Code
CFD	Computational Fluid Dynamic
$V_{NE}$	Never Exceed Speed
$R$	Blade radius
$r$	Radial distance to blade element
$\frac{r}{R}$	Radial station
$\theta$	Blade section pitch
$\sigma$	Solidity $bc/\pi R^2$
$\theta_0$	Blade pitch angle at hub
$\theta_1$	Difference between hub and tip pitch angle
$\rho$	Mass density of air
$V$	True speed of helicopter along flight path
$\Omega$	Rotor angular velocity
$v$	Induced inflow velocity at rotor
$\mu$	Tip speed ratio $(V \cos \alpha / \Omega R)$
$\lambda$	Inflow ratio $(V \sin \alpha - v / \Omega R)$
$\psi$	Blade azimuth angle

$U_T$	Component at blade element of resultant velocity perpendicular to blade-span axis
$U_P$	Component at blade element of resultant velocity perpendicular both to blade-span axis and $U_T$
$U_R$	Component at blade element of resultant velocity parallel to blade-span axis and perpendicular to $U_T$
$\phi$	Inflow angle at the blade element in plane perpendicular to blade-span axis $(\tan^{-1} U_P/U_T)$
$\alpha_r$	Blade element angle of attack $(\theta + \phi)$
$c_l$	Section lift coefficient
$a$	Slope of curve of section lift coefficient against section angle of attack
$\beta$	Blade flapping angle at particular azimuth position
$a_0$	Constant term in Fourier series that express $\beta$ (radians); hence, the coning angle
$a_n$	Coefficient of $\cos n\psi$ in expression for $\beta$
$b_n$	Coefficient of $\sin n\psi$ in expression for $\beta$
$\theta$	Blade pitch angle at particular azimuth position
$A_0$	Constant term in Fourier series that express $\theta$ (radian); hence, the mean pitch angle representative radius
$A_n$	Coefficient of $\cos n\psi$ in expression for $\theta$
$B_n$	Coefficient of $\sin n\psi$ in expression for $\theta$
$c$	Chord of the aerofoil
$f$	Equivalent flat plate area
$A$	Area of disc
$A_b$	Area of blades
$T$	Thrust
$g$	Gravity
$a$	Lift curve slope
$\alpha_{TTP}$	Tip path angle of attack

$a_0$	Coning angle
$y^+$	Distance of the first cell away from the wall
$u_T$	Friction velocity
$\mu$	Fluid viscosity
$\dot{S}_m$	Mass added to the continuous phase from the dispersed second phase
$p$	Static pressure
$\tau$	Stress tensor
$\vec{pg}$	Gravitational body force
$\vec{F}$	External body force
$k_{eff}$	Effective conductivity
$k_t$	Turbulent thermal conductivity
$\vec{J}_j$	Diffusion flux of species j
$h$	Enthalpy
$Y_j$	Mass fraction of species j
$k$	Turbulent kinetic energy
$\omega$	Specific dissipation rate
$\Gamma_k$	Effective diffusivity of $k$
$\Gamma_\omega$	Effective diffusivity of $\omega$
$Y_k$	Dissipation of $k$ due to turbulence
$Y_\omega$	Dissipation of $\omega$ due to turbulence
$\tilde{G}_k$	The generation of turbulence kinetic energy due to mean velocity gradients.
$G_\omega$	The generation of $\omega$ .
$D_\omega$	The cross-diffusion term



$\sigma_k$	Turbulent Prandtl numbers for $k$
$\sigma_\omega$	Turbulent Prandtl numbers for $\omega$
$S$	Strain rate magnitude



**LIST OF APPENDICES**

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## CHAPTER 1

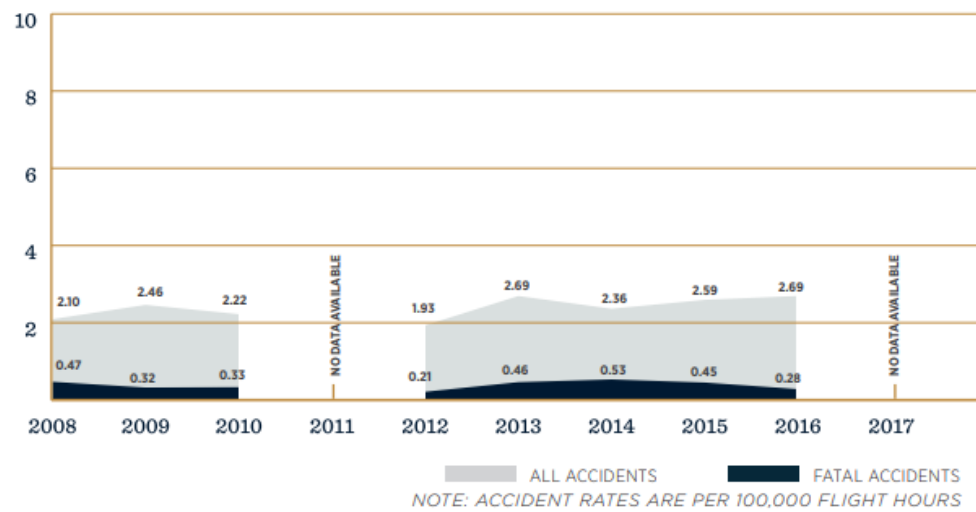
### INTRODUCTION

#### 1.1 Background study

The helicopter is a unique transport that is designed to perform at various flight maneuvers such as hovering as well as forward, rearward, sideward and vertical translations. Helicopters do not require a runway for take-off and landing and are also able to land almost everywhere. Thus, helicopters are a versatile mode of vehicle and are extensively used for missions such as air patrol, search and rescue (SAR), medical rescue missions, high-risk military missions, transportation to rural places or offshore missions and other missions of high risks.

However, the helicopter is dangerous compared to the fixed-wing. This was proven in the Report of General Aviation (GA) Accidents by AOPA Air Safety Institute (ASI) [1] as depicted in Figure 1-1. The accident rates of the helicopter increased rapidly compared to that of the fixed-wing, which declined severely from 2008 until 2016. From the years 2012 to 2016, the accident rate of the helicopter is higher than that of the fixed-wing. To be precise, in 2016, the highest rate of accidents for every 100,000 flight hours belongs to the helicopter, which is 2.69, whereas that of the fixed-wing was around 1.92. In other words, the risk of a helicopter crash versus that of the fixed-wing is about 29% greater per 100,000 hours in the air.

## COMMERCIAL HELICOPTER



## COMMERCIAL FIXED-WING

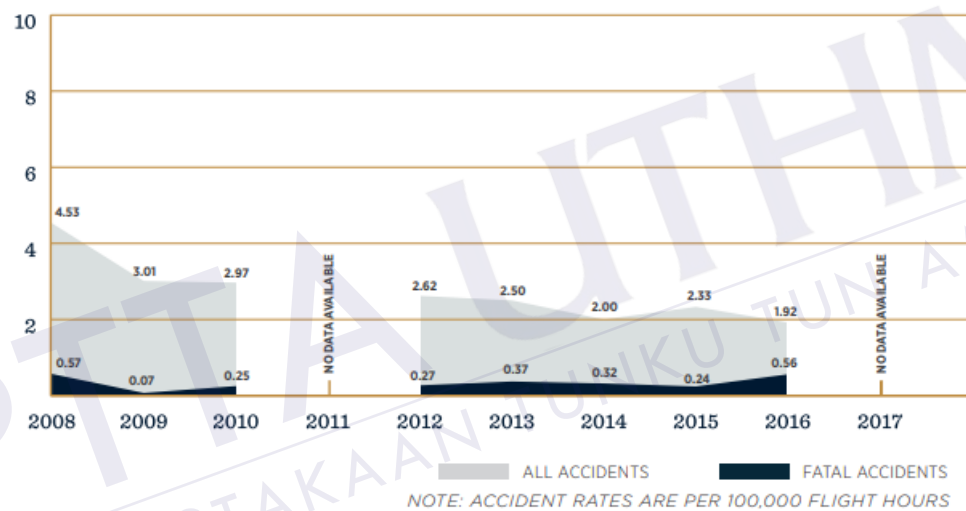


Figure 1-1: Commercial helicopter and fixed-wing accident rate from 2008 to 2017 [1].

There are four major factors shown in Figure 1-2 that contribute to the helicopter accidents, including the lack of engine airworthiness, non-engine airworthiness failures, maintenance-related failures and lastly, pilot errors/unknown causes [2]. These factors are based on 47 series of helicopter accidents for the period of 1947 through 1996. From these factors, the majority of accidents were due to pilot error/unknown causes. The pilot error refers to the U.S. rotorcraft accident data and statistics by the International Helicopter Safety Team (IHST), under Federal Aviation Administration (FAA) [3]. Reports show that the pilot error is divided into two main categories, namely pilot judgement/action problems and pilot situational awareness problems. The pilot awareness problems contribute to 34% of helicopter accidents

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